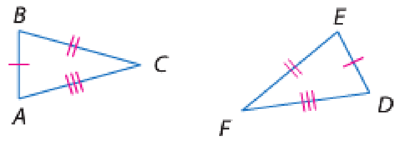
Postulate 4.1 Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

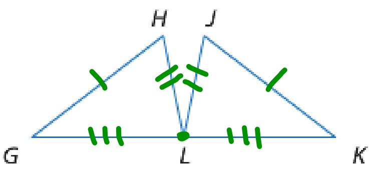
Example If **S**ide $\overline{AB} \cong \overline{DE}$,
Side $\overline{BC} \cong \overline{EF}$, and
Side $\overline{AC} \cong \overline{DF}$,
then $\triangle ABC \cong \triangle DEF$.



Example 1: Write a two-column proof.

Given: $\overline{GH} \cong \overline{KJ}$, $\overline{HL} \cong \overline{JL}$, and L is the midpoint of $\overline{GK}$. → implies $\overline{GL} \cong \overline{KL}$

Prove: $\triangle GHL \cong \triangle KJL$



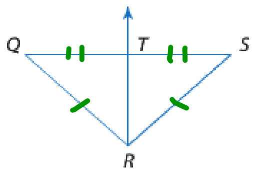
STATEMENTS	REASONS
1. $\overline{GH} \cong \overline{KJ}$, $\overline{HL} \cong \overline{JL}$, L is the midpoint of $\overline{GK}$.	1. Given
2. $\overline{GL} \cong \overline{KL}$	2. Def. of midpoint
3. $\triangle GHL \cong \triangle KJL$	3. SSS

Example 2: Write a two-column proof.

Given: $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$. $\overline{RT}$ bisects $\overline{QS}$ at point T.

↳ implies $\overline{QT} \cong \overline{ST}$

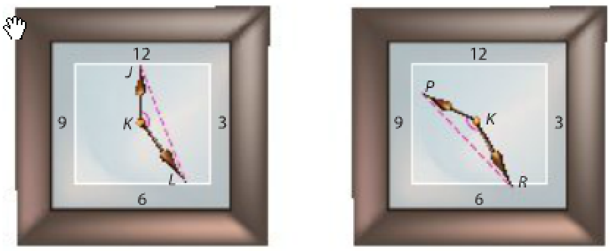
Prove: $\triangle QTR \cong \triangle STR$



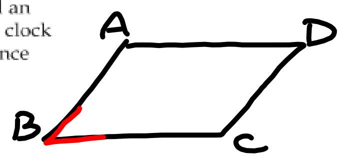
$\overline{RT}$ is a shared side → it corresponds to itself. → $\overline{RT} \cong \overline{RT}$

STATEMENTS	REASONS
1. $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$, $\overline{RT}$ bisects $\overline{QS}$ at T.	1. Given
2. $\overline{QT} \cong \overline{ST}$	2. Def. of segment bisector
3. $\overline{RT} \cong \overline{RT}$	3. Reflexive property
4. $\triangle QTR \cong \triangle STR$	4. SSS

2 SAS Postulate The angle formed by two adjacent sides of a polygon is called an **included angle**. Consider included angle JKL formed by the hands on the first clock shown below. Any time the hands form an angle with the same measure, the distance between the ends of the hands $\overline{JL}$ and $\overline{PR}$ will be the same.



$\triangle PKR \cong \triangle JKL$



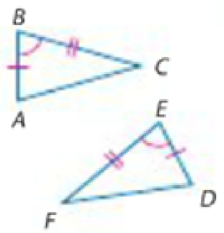
$\angle ABC$ is the included angle between $\overline{AB}$ and $\overline{BC}$.

Any two triangles formed using the same side lengths and included angle measure will be congruent. This illustrates the following postulate.

Postulate 4.2 Side-Angle-Side (SAS) Congruence Postulate

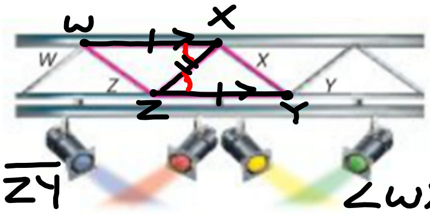
Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

Example If Side $\overline{AB} \cong \overline{DE}$,
Angle $\angle B \cong \angle E$, and → included angles
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Example 3:

LIGHTING The scaffolding for stage lighting shown appears to be made up of congruent triangles. If $\overline{WX} \cong \overline{YZ}$ and $\overline{WX} \parallel \overline{ZY}$, write a two-column proof to prove that $\triangle WXZ \cong \triangle YZX$.



Given: $\overline{WX} \cong \overline{YZ}$, $\overline{WX} \parallel \overline{ZY}$

Prove: $\triangle WXZ \cong \triangle YZX$

$\angle WXZ \cong \angle XZY$

$\overline{ZX} \cong \overline{ZX}$

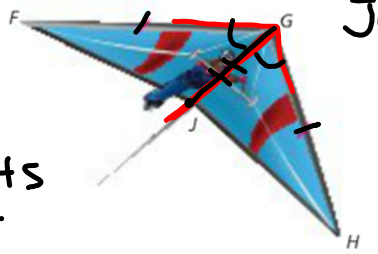
STATEMENTS	REASONS
1. $\overline{WX} \cong \overline{YZ}$, $\overline{WX} \parallel \overline{ZY}$	1. Given
2. $\angle WXZ \cong \angle XZY$	2. Alternate Interior Angles Thm.
3. $\overline{ZX} \cong \overline{ZX}$	3. Reflexive Property
4. $\triangle WXZ \cong \triangle YZX$	4. SAS

$\angle FGJ \cong \angle HGJ$

$\overline{JG} \cong \overline{JG}$

Example 4:

EXTREME SPORTS The wings of the hang glider shown appear to be congruent triangles. If $\overline{FG} \cong \overline{GH}$ and $\overline{JG}$ bisects $\angle FGH$, prove that $\triangle FGJ \cong \triangle HGJ$.



Given: $\overline{FG} \cong \overline{GH}$, $\overline{JG}$ bisects $\angle FGH$.

Prove: $\triangle FGJ \cong \triangle HGJ$

STATEMENTS	REASONS
1. $\overline{FG} \cong \overline{GH}$, $\overline{JG}$ bisects $\angle FGH$	1. Given
2. $\angle FGJ \cong \angle HGJ$	2. Def. of Angle Bisector
3. $\overline{JG} \cong \overline{JG}$	3. Reflexive Property
4. $\triangle FGJ \cong \triangle HGJ$	4. SAS